Mathematics 8 Unit 1 – Transformations, Congruence, and Similarity

(5 weeks)

Unit Overview: Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems.

Content Standards

Understand congruence and similarity using physical models, transparencies, or geometry software.

MCC.8.G.1 Verify experimentally the properties of rotations, reflections, and translations: *(ITBS) (PSAT-M6)*

a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure.

c. Parallel lines are taken to parallel lines.

MCC.8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. *(ITBS)*

MCC.8.G.3 Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates. *(ITBS)*

CC.8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. *(ITBS)*

MCC.8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the three angles appear to form a line, and give an argument in terms of transversals why this is so. (ITBS)*

TRANSITIONAL STANDARD-TEACH 2012-13

MCC.7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. *(ITBS)(PSAT-M6)*

Standards for Mathematical Practice:

1 Make sense of complex problems and persevere in solving them.

3 Construct viable arguments and critique the reasoning of others.

Diagnostic: Prerequisite Assessment

Standards for Mathematical Practice (1, 3)

EQ: How do mathematically proficient students approach problem solving? (MP1) How do you know if you have a convincing argument? (MP3)

Learning Targets:

I can …

- use Polya's four steps of problem solving to work through word problems. (MP1)
- demonstrate and organize my work using Polya's four steps of problem solving (MP1)
- Plan a solution pathway rather than simply jumping into a solution attempt. (MP1)
- Monitor and evaluate my progress and change course if necessary. (MP1)
- Check answers to problems using a different method. (MP1)
- Understand the approaches of others to solving problems. (MP1)
- use definitions and previously established causes/effects (results) in constructing arguments (MP3)
- make conjectures and use counterexamples to build a logical progression of statements to explore and support their ideas (MP3)
- communicate and defend mathematical reasoning using objects, drawings, and diagrams (MP3)
- listen to or read the arguments of others (MP3)
- decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments (MP3)

Concept Overview:

SMP 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. High school students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

In his book, *How to Solve It*, George Polya describes his study of effective problem solvers. He surmised that if he discovered commonalities among good problem solvers, that he could teach others to become good problem solvers. His hypothesis turned out to be correct. He found that by explicitly teaching students the four steps of problem solving, their problem solving abilities actually improved. The four steps are as follows:

1. Understand the problem

- What are you asked to find out or show?
- Can you draw a picture or diagram to help you understand the problem?
- Can you restate the problem in your own words?
- Can you work out some numerical examples that would help make the problem clearer?

2. Devise a plan

A partial list of Problem Solving Strategies include:

Guess and check	Solve a simpler problem
Make an organized list	Experiment
Draw a picture or diagram	Act it out
Look for a pattern	Work backwards
Make a table	Use deduction
Use a variable	Change your point of view

3. Carry out the plan

- Carrying out the plan is usually easier than devising the plan
- Be patient – most problems are not solved quickly nor on the first attempt
- If a plan does not work immediately, be persistent
- Do not let yourself get discouraged
- If one strategy isn’t working, try a different one

4. Look back (reflect)

- Does your answer make sense? Did you answer all of the questions?
- What did you learn by doing this?
- Could you have done this problem another way – maybe even an easier way?

MP 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Resources:

Teaching Problem Solving Strategies
Problem Solving Rubric
MP1 Inside Mathematics Website
MP3 Inside Mathematics Website

Understanding Congruence and Similarity

Understand congruence and similarity using physical models, transparencies, or geometry software.

MCC.8.G.1 Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.

M**CC.8.G.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

MCC.8.G.3 Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.

MCC.8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

EQ: How can physical models increase our understanding of congruent and similar figures?

Learning Targets:

I can …

- verify – by measuring and comparing lengths, angle measures, and parallelism of a figure and its image – that after a figure has been translated (reflected, and/or rotated), corresponding lines and line segments remain the same length, corresponding angles have the same measure, and corresponding parallel lines remain parallel. **(G.1)**
- explain how transformations can be used to prove that two figures are congruent. **(G.2)**
- perform a series of transformations (reflections, rotations, and/or translations) to prove or disprove that two given figures are congruent. **(G.2)**
- describe the changes occurring to the x – and y – coordinates of a figure after a translation (reflection, rotation and/or dilation). **(G.3)**

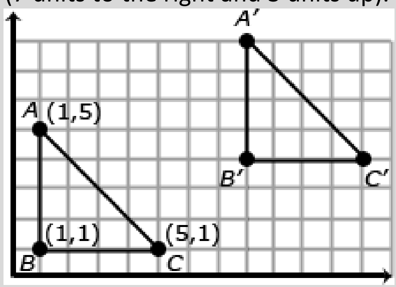
Concept Overview: A major focus in Grade 8 is to use knowledge of angles and distance to analyze two- and three-dimensional figures and space in order to solve problems. This cluster interweaves the relationships of symmetry, transformations, and angle relationships to form understandings of similarity and congruence. Inductive and deductive reasoning are utilized as students forge into the world of proofs. Informal arguments are justifications based on known facts and logical reasoning. Students should be able to appropriately label figures, angles, lines, line segments, congruent parts, and images (primes or double primes). Students are expected to use logical thinking, expressed in words using correct terminology. They are **NOT** expected to use theorems, axioms, postulates or a formal format of proof as in two-column proofs.

Transformational geometry is about the effects of rigid motions, rotations, reflections and translations on figures. Initial work should be presented in such a way that students understand the concept of each type of transformation and the effects that each transformation has on an object before working within the coordinate system. For example, when reflecting over a line, each vertex is the same distance from the line as its corresponding vertex. This is easier to visualize when not using regular figures. Time should be allowed for students to cut out and trace the figures for each step in a series of transformations. Discussion should include the description of the relationship between the original figure and its image(s) in regards to their corresponding parts (length of sides and measure of angles) and the description of the movement, including the attributes of transformations (line of symmetry, distance to be moved, center of rotation, angle of rotation and the amount of dilation). The case of distance – preserving transformation leads to the idea of congruence.

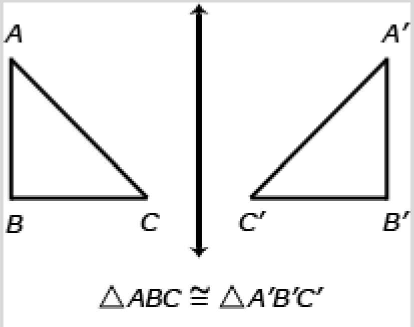
In a translation, every point of the pre-image is moved the same distance and in the same direction to form the image. A reflection is the “flipping” of an object over a line, known as the “line of reflection”. A rotation is a transformation that is performed by “spinning” the figure around a fixed point known as the center of rotation. The figure may be rotated clockwise or counterclockwise. Students use compasses, protractors and ruler or technology to explore figures created from translations, reflections and rotations. Characteristics of figures, such as lengths of line segments, angle measures and parallel lines are explored before the transformation (pre-image) and after the transformation (image). Students understand that these transformations produce images of exactly the same size and shape as the pre-image and are known as rigid transformations. Students need multiple opportunities to explore the transformation of figures so that they can appreciate that points stay the same distance apart and lines stay at the same angle after they have been rotated, reflected, and/or translated. Students are not expected to work formally with properties of dilations until high school

A dilation is a transformation that moves each point along a ray emanating from a fixed center, and multiplies distances from the center by a common scale factor. In dilated figures, the dilated figure is *similar* to its pre-image.

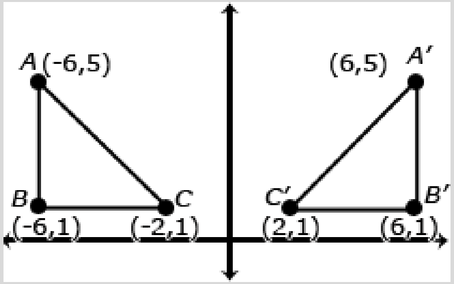
A translation is a transformation of an object that moves the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is *congruent* to its pre-image. ΔABC has been translated 7 units to the right and 3 units up. To get from A (1,5) to A' (8,8), move A 7 units to the right (from x = 1 to x = 8) and 3 units up (from y = 5 to y = 8). Points B + C also move in the same direction (7 units to the right and 3 units up).



Reflection: A reflection is a transformation that flips an object across a line of reflection (in a coordinate grid the line of reflection may be the x or y axis). In a rotation, the rotated object is *congruent* to its pre-image.

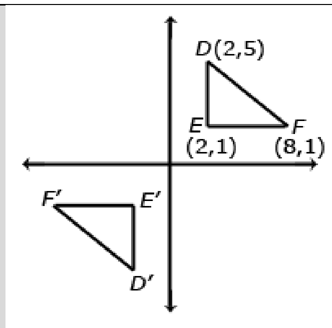


When an object is reflected across the y axis, the reflected x coordinate is the opposite of the pre-image x coordinate.



Rotation: A rotated figure is a figure that has been turned about a fixed point. This is called the center of rotation. A figure can be rotated up to 360˚. Rotated figures are congruent to their pre-image figures.

"Consider when ΔDEF is rotated 180˚ clockwise about the origin. The coordinates of ΔDEF are D(2,5), E(2,1), and F(8,1). When rotated 180˚, $\Delta D'E'F'$ has new coordinates D'(-2,-5), E'(-2,-1) and F'(-8,-1). Each coordinate is the opposite of its pre-image.



Congruent figures have the same shape and size. Translations, reflections and rotations are examples of rigid transformations. A rigid transformation is one in which the pre-image and the image both have exactly the same size and shape since the measures of the corresponding angles and corresponding line segments remain equal (are congruent).
Students examine two figures to determine congruency by identifying the rigid transformation(s) that produced the figures. Students recognize the symbol for congruency (≅) and write statements of congruency.

In addition to congruency, students are introduced to similarity and similar figures in this unit. Students understand similar figures have angles with the same measure and sides that are proportional. Similar figures are produced from dilations. Students describe the sequence that would produce similar figures, including the scale factors. Students understand that a scale factor greater than one will produce an enlargement in the figure, while a scale factor less than one will produce a reduction in size.

Vocabulary:

congruent: Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

***similar figures*:** Figures that have the same shape but not necessarily the same size with congruent corresponding angles and proportional corresponding sides. One figure can be obtained from the other by uniformly scaling (enlarging or shrinking), possibly with additional translation, rotation and reflection.

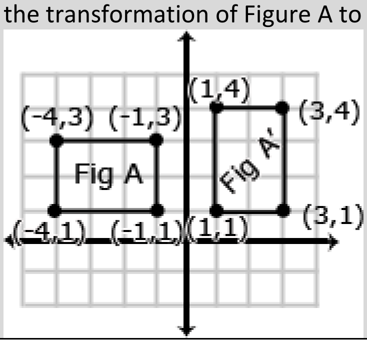
transformation: The mapping, or movement, of all the points of a figure in a plane according to a common operation.

reflection: A transformation that "flips" a figure over a line of reflection

translation: A transformation that "slides" each point of a figure the same distance in the same direction.

rotation: A transformation that turns a figure about a fixed point through a given angle and a given direction.

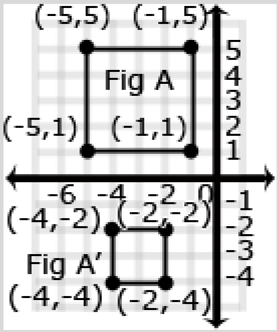
Sample Problem(s): **Solutions to Sample Problems**
Sample Problem 1 (MCC.8.G.2)
Is Figure A congruent to Figure A'? If not, describe how you know. If so, describe the sequence of transformations that results in the transformation of Figure A to Figure A'.



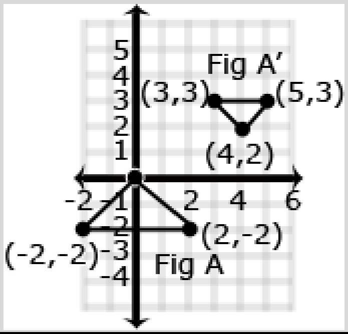
Sample Problem 2 (MCC.8.G.3)
The vertices of Triangle A are (1,0), (1,1), (0,0) and Triangle A’ are (2,1), (2,2), (3,1). Describe the series of transformations performed on Triangle A that result in Triangle A’.

Sample Problem 3 (MCC.8.G.4)
Which of the following transformations will result in a similar figure? Explain why the other choices will not result in a similar figure.
A) (3*x*, 2*y*)
B) (-*x*+2, *y*-2)
C) (5*x*, *y*+5)
D) (3*x*, *x*+*y*)

Sample Problem 4 (MCC.8.G.4)
Is Figure A similar to Figure A’? Explain how you know.



Sample Problem 5 (MCC.8.G.3)
Describe the sequence of transformations that results in the transformation of Figure A to Figure A’.



Standard	Topic	Resources	Teacher Notes
MCC.8.G.1	Verifying properties of transformations (translations, reflections, rotations)	**Pearson 3** 3-6 pg 136-139 3-7 pg 141-144 3-8a pg 145 3-8 pg 146-150 **Pearson 2** 10-5 pg 510-513 10-6 pg 514-518 10-7 pg 519-522 **Holt 3** 7-7 pg 358-362	Transformations Model Lesson Model Lesson Activity 1 (discovery) Model Lesson Activity 1 Answers Model Lesson Activity 2 (reflection) Model Lesson Activity 2 Answers Model Lesson Activity 3 (translation & reflection)

		Math 8 Unit 1 Transformation Website 1 (U) Math 8 Unit 1 Transformation Interactive Web Activity 2 (U) Math 8 Unit 1 Transformation Interactive Web Activity 3 (M) Math 8 Unit 1 Transformation Video (S)	Model Lesson Activity 3 Answers Student Misconceptions: Students often confuse situations that require adding with multiplicative situations in regard to scale factor. Providing experiences with geometric figures and coordinate grids may help students visualize the difference. Probing questions &Differentiation Strategies: Literacy Strategy: Vocabulary Anticipation Guide (U)
MCC.8.G.2	Transformational Definition of Congruence	**Pearson 2** 7-5 pg 346-349 **Holt 3** 7-6 pg 354-357 Math 8 Unit 1 Congruence Worksheet 1 (S) Math 8 Unit 1 Congruence Worksheet 2 (U) Math 8 Unit 1 Congruence Worksheet 3 (M)	Probing questions & Differentiation Strategies: Literacy Strategy: Journal Writing Congruence. (U)
MCC.8.G.3	Transformations in the coordinate plane (translations, reflections, rotations, dilations)	**Pearson 3** 3-6 pg 136-139 3-7 pg 141-144 3-8a pg 145 3-8 pg 146-150 **Pearson 2** 10-5 pg 510-513 10-6 pg 514-518 10-7 pg 519-522 **Holt 3** 7-7 pg 358-362 Math 8 Unit 1 Translation Worksheet (S) Math 8 Unit 1 Interactive Web Activity 5 (U) Math 8 Unit 1 Transformation Game (M) Log in as guest and choose "Flip-n-Slide"	Student Misconceptions: • Students may think the terms translation, reflection and rotation are interchangeable. • Clockwise and counterclockwise are the same or interchangeable terms. Probing questions & Differentiation Strategies: Cooperative Learning Strategy: Transformational Geometry Mix N Match Literacy Strategy: Three Gears of Prereading: Skimming, Scanning, Sampling (U)

MCC.8.G.4	Similarity	**Pearson 3** 3-6 pg 136-139 3-7 pg 141-144 3-8a pg 145 3-8 pg 146-150 **Pearson 2** 10-5 pg 510-513 10-6 pg 514-518 10-7 pg 519-522 Math 8 Unit 1 Scale Drawings Interactive Web Activity (U) Math 8 Unit 1 Similarity Website (M)	Student Misconceptions and common errors: • Students confuse congruence with similarity. • Students sometimes confuse the different kinds of notation used with similarity, such as =∼ and ∼. • A common misconception is that when the dimensions of an object are doubled, the area is doubled, too. • Students may confuse additive thinking vs. multiplicative thinking. *For example,* suppose there were two similar rectangles with a pair of corresponding side lengths of 4 and 7 cm. The smaller similar rectangle has a width of 3 cm. A student may think the other corresponding side should have a length of 6 cm, because the first set of corresponding side lengths changed by 3 cm, so they might think that the other should also change by adding 3 cm. Probing questions & Differentiation Strategies: Literacy Strategy: Find the Fake (U)

Angles

Angle relationships and Measures

MCC.8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the three angles appear to form a line, and give an argument in terms of transversals why this is so. (ITBS)*

EQ: How can we determine the angle sum and exterior angle of triangles using informal arguments?

Learning Targets:

I can …

- informally prove that the sum of any triangle's interior angels will have the same measure as a straight line.
- informally prove that the sum of any polygon's exterior angles will be 360-degrees.
- make conjectures regarding the relationships and measurements of the angles created when two parallel lines are cut by a transversal.
- apply proven relationships to establish minimal properties to justify similarity.

Concept Overview: In Grade 7, students develop an understanding of the special relationships of angles and their measures (complementary, supplementary, adjacent, vertical). Now, the focus is on learning the about the sum of the angles of a triangle and using it to, find the measures of angles formed by transversals (especially with parallel lines), or to find the measures of exterior angles of triangles and to informally prove congruence. Students use exploration and deductive reasoning to determine relationships that exist between a) angle sums and exterior angle sums of triangles, b) angles created when parallel lines are cut by a transversal, and c) the angle-angle criterion for similarity of triangle. Students construct various triangles and find the measures of the interior and exterior angles. Students make conjectures about the relationship between the measure of an exterior angle and the other two angles of a triangle (the measure of an exterior angle of a triangle is equal to the sum of the measures of the other two interior angles) and the sum of the exterior angles (360°). Using these relationships, students use deductive reasoning to find the measure of missing angles. Students construct parallel lines and a transversal to examine the relationships between the created angles. Students recognize vertical angles, adjacent angles and supplementary angles from 7th grade and build on these relationships to identify other pairs of congruent angles. Using these relationships, students use deductive reasoning to find the measure of missing angles. Students construct various triangles having line segments of different lengths but with two corresponding congruent angles. Comparing ratios of sides will produce a constant scale factor, meaning the triangles are similar.

By using three copies of the same triangle labeled and placed so that the three different angles form a straight line, students can:
• explore the relationships of the angles,
• learn the types of angles (interior, exterior, alternate interior, alternate exterior, corresponding, same side interior, same side exterior), and
• explore the parallel lines, triangles and parallelograms formed.
Further examples can be explored to verify these relationships and demonstrate their relevance in real life.

Transition Standard: In previous grades, students have studied angles by type according to size: acute, obtuse and right, and their role as an attribute in polygons. Now angles are considered based upon the special relationships that exist among them: supplementary, complementary, vertical and adjacent angles. Provide students the opportunities to explore these relationships first through measuring and finding the patterns among the angles of intersecting lines or within polygons, then utilize the relationships to write and solve equations for multi-step problems.

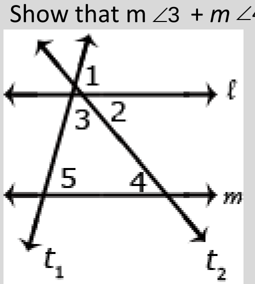
Vocabulary:
Interior angles: The inside angles of a polygon.

Exterior angle: the angles that are adjacent to the interior angles and formed on the outside.

Parallel Lines: Lines in the same plane that never intersect.

Transversal: A line that intersects two or more lines at different points.

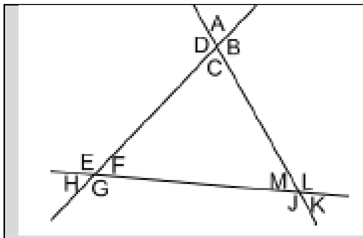
Sample Problem(s): Solutions to Sample Problems
Sample Problem 6 MCC.8.G.5
Show that m $\angle 3$ + $m\angle 4$ + $m\angle 5$ = 180˚ if l and m are parallel lines and t_1 & t_2 are transversals.



Solution:
$\angle 1 + \angle 2 + \angle 3$ = 180˚. Angle 1 and Angle 5 are congruent because they are corresponding angles ($\angle 5 \cong \angle 1$). $\angle 1$ can be substituted for $\angle 5$.
$\angle 4 \cong \angle 2$: because alternate interior angles are congruent.
$\angle 4$ can be substituted for $\angle 2$
Therefore m $\angle 3$ + m $\angle 4$ + m $\angle 5$ = 180˚

Sample Problem 7 (MCC.8.G.5)
The streets 400 E and 900 E run north and south. Euclid Drive cuts both of these streets at an angle from SE to NW. Pythagoras Way passes through all three streets SW to NE. Are all possible triangles created by the intersection of the streets similar? Justify.

Given $m\angle B$=102° and $m\angle L$=120°, find every other angle measure, explaining how you found each.



Standard	Topic	Resources	Teacher Notes
MCC.8.G.5	Angle relationships	**Pearson 3** 7-2 pg 307-310 **Holt 3** 7-2 pg 330-333 7-3 pg 336-340 Math 8 Unit 1 Angle Sums Interactive Web Activity (U)	Angle Relationship Model Lesson Model Lesson resource Model Lesson resource key Student Misconceptions: • Students are unfamiliar with the symbolic notation used to identify angles and their measures. • Students may use the point of intersection to name all angles formed by a pair of intersecting lines. *For example,* all angles formed by a pair of lines that intersect at point A may be referred to as angle A. • Students confuse the terms *supplementary* and *complementary.* • Students believe that complementary and supplementary angles must be adjacent. • Students believe that all adjacent angles are either complementary or supplementary. • Students may incorrectly identify vertical angles. • Students will think that angles are congruent when there are no parallel lines present. Emphasize that angles (alternate exterior, alternate interior, corresponding, and adjacent angles) can only be congruent when they are formed by parallel lines being intersected by a transversal. • Students may be confused when the transversal is slanted differently. Exploring examples where the transversal is increasing from left to right and decreasing from left to right will help with this confusion. Probing questions & Differentiation Strategies: Cooperative Learning Strategy: Vocabulary Review Activity Literacy Strategy: Vocabulary strategy - foldable (U)
MCC.7.G.5	Solving problems	**Pearson 3** 7-1 pg 303-306	

	involving angles	• Math 8 Unit 1 Exterior Angle Worksheet (U)	Literacy Strategy: SQR3 for Math (U)
Unit 1 Summative Assessment Question Bank Unit 1 Summative Assessment Question Bank Solutions			